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TENSILE FATIGUE STRENGTH OF BRITTLE MATERIALS

R. SEDLACEK

Stanford Research Institute

TECHNICAL REPORT AFML-TR-66-245

NOVEMBER 1968

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AFML-TR-66-245

TENSILE FATIGUE STRENGTH OF BRITTLE MATERIALS

R. SEDLACEK

FOREWORD

This work was performed by Stanford Research Institute under USAF Contract No. AF 33(657)-10600. The contract was initiated under Project No. 7350, "Refractory Inorganic Nonmetallic Materials," Task No. 735003, "Refractory Inorganic Nonmetallic Materials, Theory and Mechanical Phenomena." The work was under the administrative direction of the Air Force Materials Laboratory, Wright-Patterson Air Force Base, Ohio. Mr. G. R. Atkins was the Project Engineer.

This report covers work conducted from 2 February 1965 to 2 February 1966.

This program was under the specific direction of R. Sedlacek and the general direction of F. A. Halden.

Manuscript released by the author in June 1966 for publication as a technical report.

This technical report has been reviewed and is approved.

W. J. TRAPP
Chief, Strength and Dynamics Branch
Metals and Ceramics Division
Air Force Materials Laboratory

ABSTRACT

The cyclic fatigue behavior of a high-alumina body was investigated under conditions of cyclic tensile stress by using hydraulically expanded cylindrical specimens. Experimental results show that the fatigue strength of alumina decreases with increasing maximum stress and is influenced by the value of the stress ratio. Specimens which reached the arbitrary fatigue life of 24 hours (345,000 cycles) were subsequently stressed to failure at a rate of 10^4 psi/sec. Their average ultimate tensile strength was 32,800 psi, indicating that the cyclic stress conditions under which that fatigue life was reached did not weaken the test material and could probably be sustained indefinitely.

S-N curves and a constant life fatigue diagram for the alumina body studied were established.

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GLOSSARY

The following definitions of terms used in the text are taken from: 1965 Book of ASTM Standards, Part 30, pp. 127-129 and 585-591; Dieter, G. E., Mechanical Metallurgy, McGraw-Hill, New York, 1961, pp. 296-332; and Shigley, J. E., Mechanical Engineering Design, McGraw-Hill, New York, 1963, pp. 157-191.

Stress cycle	-	the smallest segment of the stress-time function that is repeated periodically
Fatigue life, N	-	the number of cycles of stress that a specimen sustains before failure
Maximum stress, σ_{max}	-	the stress having the highest algebraic value in the stress cycle
Minimum stress, σ_{min}	-	the stress having the lowest algebraic value in the stress cycle
Mean stress, σ_m	-	the algebraic average of the maximum and minimum stresses in one cycle $\frac{\sigma_{max} + \sigma_{min}}{2}$
Range of stress	-	the algebraic difference between the maximum and minimum stresses in one cycle $(\sigma_{max} - \sigma_{min})$
Stress amplitude, σ_a	-	one-half the range of stress $\frac{\sigma_{max} - \sigma_{min}}{2}$
Stress ratio, R	-	the ratio of the minimum stress to the maximum stress $(\sigma_{min}/\sigma_{max})$
Constant life fatigue diagram	-	a plot of a family of curves relating, for a single fatigue life (N), σ_a, σ_{max}, and σ_{min} to σ_m

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I INTRODUCTION

Previous studies performed under this contract have demonstrated that the method of tensile testing of brittle materials developed at Stanford Research Institute produces highly reproducible data and permits detection of small changes of tensile strength that originate in the manufacturing process or that result from various modes of load application.[1] During the first phase of this investigation, lot-to-lot variations in tensile strength of a high-alumina, commercial ceramic body were evaluated in the light of nonuniformity of process variables.[2] During the second phase, the tensile strength of the same alumina body was measured as a function of stress rate and gauge volume. Also, load-bearing capability under conditions of constant static stress and the effect of this stress on the ultimate tensile strength were determined.[3] This report, covering the work performed in the final phase of this contract, describes the behavior of alumina under cyclic tensile stress and the effect of this stress on ultimate tensile strength.

The main reason for this interest in the production and testing variations described is that most structural components in actual service are more likely to be exposed to various kinds of repetitive stresses than to constant static stresses or loads rising at linear rates, while most test procedures for ceramics employ only the latter type of loading.

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[1] Sedlacek, R., and F. A. Halden, "Method of Tensile Testing of Brittle Materials," Rev. Sci. Instr., 33, 298-300 (1962).

[2] Sedlacek, R., Tensile Strength of Brittle Materials, Technical Documentary Report No. ML-TDR-64-49, Air Force Materials Laboratory, Wright-Patterson Air Force Base, Ohio, March 1964.

[3] Sedlacek, R., Tensile Strength of Brittle Materials, Technical Report AFML-TR-65-129, Air Force Materials Laboratory, Wright-Patterson Air Force Base, Ohio, August 1965.

In the case of metals, it is well known that their resistance to cyclic stresses is much lower than their tensile strength determined in a single-stroke test, and a wealth of information on cyclic fatigue is available to the designer. Also, test procedures and equipment are reasonably standardized, and a meaningful comparison of data is possible. Unfortunately, little of this knowledge is directly applicable to oxide-based ceramics for various reasons, of which brittleness is perhaps the most important. It is known that, under cyclic stress, metals undergo flow and work-hardening prior to fracture; no such microstructural changes have been observed in polycrystalline ceramics. On the other hand, ceramics definitely exhibit the phenomenon of delayed fracture (static fatigue), which is nonexistent in most metals. It can therefore be safely assumed that different mechanisms control the fatigue behavior of these two basic classes of materials.

Since the scientific interest in using ceramics for structural applications does not date back very far, it is not surprising that only a scanty amount of data is available on the subject of fatigue behavior of ceramics. Inorganic glasses have received more attention, being isotropic and homogeneous materials, but it is difficult to decide how much of the information obtained on glasses can be applied to polycrystalline ceramics.

Notwithstanding the diversity of test materials and experimental procedures used, all previous studies agree on several points. First, it has been shown that the fatigue life of glasses and ceramics is highly dependent on the magnitude of applied stress. This is not too surprising if one accepts the existence of the phenomenon of delayed fracture, but the extent to which stress controls the fatigue life is unexpected. For instance, Pearson[4] has shown that the fatigue life of alumina stressed in bending can be extended from 1 sec to 10^6 sec by

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[4] Pearson, S., "Delayed Fracture of Sintered Alumina," Proc. Phys. Soc. (London) 69(B), 1293-96 (1956).

reducing the stress by 22%. It has been our experience[3] that high-density alumina can bear static tensile stresses only on the order of 60% of the single-stroke tensile strength without damaging the material and that stresses greater than this will significantly impair the residual strength of the material. Another commonly observed phenomenon is the increase of failure stress with increasing stress rate.[3,5] The existence of delayed fracture and the dependence of strength on stress rate have been described as the effect of a stress-enhanced chemical attack by atmospheric constituents (H_2O, CO_2) acting on the tip of Griffith-type flaws.

Cyclic fatigue of ceram's and glasses has not received the attention which it deserves, and some of the few results published are highly contradictory. Perhaps the most detailed examination of the behavior of alumina under conditions of cyclic fatigue was made by Williams,[6] who reached the conclusion that the effect of cyclic stresses is more severe than that of static stresses. On the other hand, Gurney and Pearson, working with glass, found that this material does not weaken much faster under cylcic stresses than it does under static loading.[7]

In all cyclic fatigue studies on ceramics and glasses reported to date, various rotating-cantilever arrangements were used. This means that the test materials were exposed to tensile as well as compressive stresses of equal magnitude. In the work described in this report, test specimens were cycled in tension only, so that if there is any effect of compressive stresses on the cyclic fatigue behavior of alumina, this effect can be neglected in the interpretation of results.

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[5] Weil, N. A., Studies of the Brittle Behavior of Ceramic Materials, Technical Documentary Report No. ASD-TR-61-628, Air Force Materials Laboratory. Wright-Patterson Air Force Base, Ohio, April 1962.

[6] Williams, L. S., "Stress-Endurance of Sintered Alumina," Trans. Brit. Ceram. Soc., 55, 287-312 (1956).

[7] Gurney, C., and S. Pearson, "Fatigue of Mineral Glass under Static and Cyclic Loading," Proc. Roy. Soc. (A), 192, 537 (1948).

II SUMMARY

The endurance of Al-995 alumina to cyclic tensile stresses at a frequency of 4 cps was evaluated. In the experimental arrangement, specimens were divided into three groups, for each of which the stress ratio, $R = \sigma_{min}/\sigma_{max}$, was a different constant (i.e., 0.14 for Group A, 0.33 for Group B, and 0.78 for Group C).

Results show that the resistance of alumina to cyclic tensile stresses decreases rapidly with increasing maximum stress, and that it is--at least at lower stress levels--an inverse function of the stress range. The maximum tensile stresses for the arbitrary fatigue life of 345,000 cycles were 15,800 psi for Group A, 16,300 psi for Group B, and 16,800 psi for Group C.

S-N curves were obtained from which a diagram of alternating stress versus mean stress was constructed. The modified Goodman line joining the points of most probable combinations of stresses at the 345,000 cycle fatigue life intersects the abscissa in the vicinity of 20,000 psi; this is a somewhat higher value than the 18,000 psi obtained previously under conditions of constant static stress.

Specimens which reached the arbitrary fatigue life were subsequently loaded in tension to failure at the rate of 10^4 psi/sec. Their average ultimate tensile strength was 32,900 $\pm$ 2,400 psi, indicating that little or no damage was caused by cyclic stress combinations under which the 24-hour limit was reached.

The capability of the alumina specimens used in this study to sustain constant static stresses at the 26,800 psi and 23,900 psi levels was evaluated. The average life spans were 7.7 and 133 seconds, respectively--in reasonable agreement with previous results obtained on a different group of specimens having the same nominal composition.

III EXPERIMENTAL STUDIES

A. Material

Test specimens used in this study were a high-alumina commercial body (Al-995) produced by the Western Gold and Platinum Company, Belmont, California. All specimens originated from the same batch of raw material and were processed as uniformly as possible. Isostatically formed hollow cylinders were bored, turned, and finally sliced into rings, so that a minimum of grinding was required after firing. All specimens were fired simultaneously, in close setting, in a large gas-fired kiln under a normal production schedule. The grinding procedure employed by the manufacturer was as follows: first the rings were faced on a Blanchard grinder to assure maximum parallelism; then they were arranged in stacks and centerlessly ground to the final outside diameter; finally, with the stacks contained in a special Plexiglas collet, the inside diameter was finished on a cylindrical grinder, and both end pieces of each stack were rejected.

In final inspection it was found that about half of all specimens had unacceptably jagged edges. These specimens, therefore, had to be refaced and shortened by 0.020 inch. The final dimensions of the test specimens were as follows:

Outside diameter:	2.200 ± 0.001 inch
Inside diameter:	2.000 ± 0.001 inch
Length:	0.250 or 0.230 ± 0.0005 inch

In the course of work, air gauges became available for measuring, without direct contact, the inside diameter and wall thickness of cylinders to a few hundred-thousandths of an inch. These measurements indicated that some grinding problems exist. For instance, it was found that the inside surfaces of all specimens measured were slightly elliptical, with semiaxes 90° apart. The source of this deviation from a true circle is

unknown, but it can be speculated that the collets used in internal grinding exert on the stacked specimens a compressive force higher in the plane of one diameter than in the plane perpendicular to it. On the other hand, the wall thickness varied less in magnitude than the inside diameter, but much more erratically, as if the outside walls were wavy. No specimens were rejected because of variations of wall thickness exceeding the specified tolerances, but a considerable number of them were discarded for variations of inside diameter beyond tolerances.

Test results showed no correlation between strength and the geometry or weight of the specimens. Only in one instance was a fracture plane observed exactly at the intersection of the long semiaxis and the specimen wall, i.e., at the locus of maximum bending moment.

The surface finish, measured with a Brush Instruments Model MS-1000 Surfindicator, was 30 to 60 microinches RMS. This is slightly rougher than was measured on similar specimens in previous years (20 to 35 microinches RMS). However, the edges of the specimens appeared smoother than those of specimens purchased previously. The microstructure showed an average grain size of 20 to 30 microns, and the pycnometric density was 3.850 g/cc. Both of these values are identical to previously measured values. A typical microstructure of the specimens tested is shown in Fig. 1. Emission spectrochemical analysis was performed by the American Spectrographic Laboratories, Inc., San Francisco; the following amounts of oxides of the elements were found:

Al - Principal constituent
Si - 0.75%
Mg - 0.5%
Mn - 0.001%
Ca - 0.015%
Fe - 0.15%
Ti - 0.01%
Cu - 0.03%
Cr - 0.003%
Ba - 0.005%

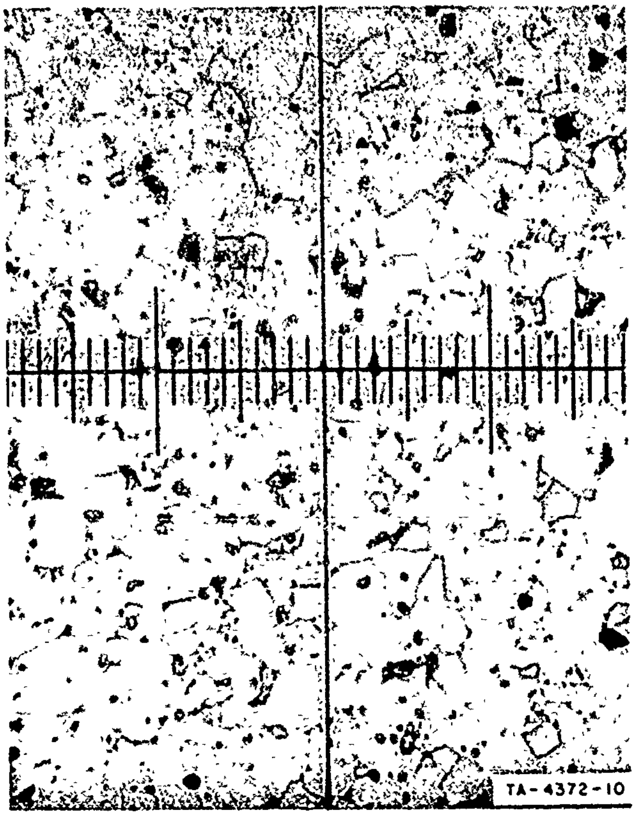


Figure 1. Microstructure of Al-995 Alumina (X350)

B. Apparatus

The apparatus used in this study consisted of the original specimen holder, a dead weight gauge tester, 5,000 psi and 10,000 psi capacity pressure transducers feeding their signals into an oscillograph (Visicorder 1508), and a millivolt recorder (Esterline Angus S 601S). Both transducers operate off a Wiancko power supply (Type 1-3001 carrier oscillator, Type 2-3003 demodulators, and Type 6-3004 range and balance units). All these units have been described in detail previously.[2,3] A schematic diagram of the assembled apparatus is shown in Fig. 2.

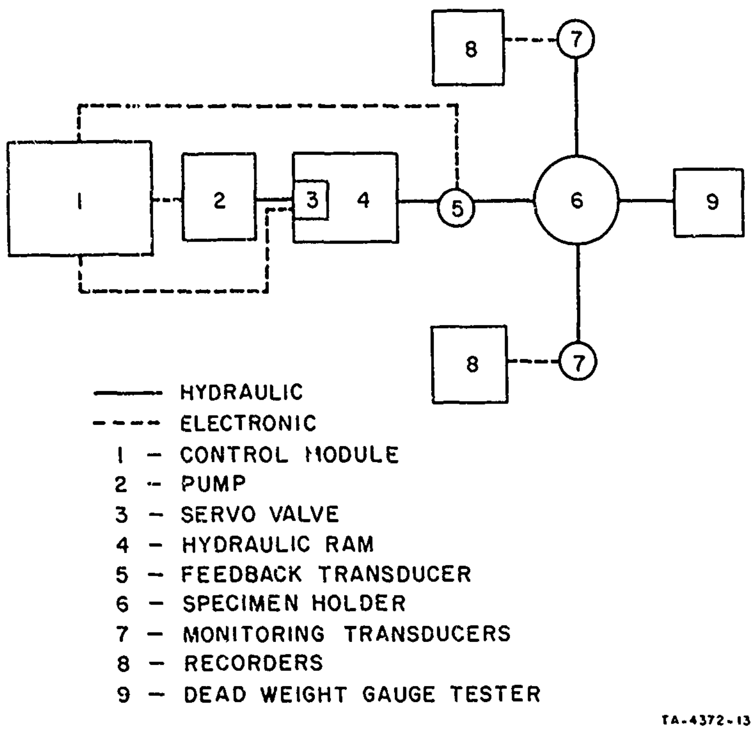


FIG. 2 BLOCK DIAGRAM OF APPARATUS

The pressure generating part of the apparatus consists of a double closed loop electrohydraulic system manufactured by the MTS Division of Research, Incorporated, Minneapolis, Minnesota. This system is made up of individual electronic units which control, through a servo valve, the flow of hydraulic oil into the low pressure end of a two-stage, differential-area-type ram. Pressure is supplied by a separate pump operating at constant pressure. The type of pressurization desired is chosen on a function generator offering the following modes of operation: ramp to fracture, ramp to hold at preset pressure levels, and various cyclic modes at frequencies from 0.001 to 1000 cps. However, the working frequency of the hydraulics is much lower and is strongly dependent on the range of pressures used. Since the pressures required in this work were such that only about 10 cps were practicable under optimum conditions, the study of cyclic fatigue of alumina was carried out using the sinusoidal mode at 4 cps.

By comparison of the pressure command signal and the actual pressure experienced by the test specimen, the main control element (Servac, Model 401-01) provides continuous corrective signals to the servo valve controlling the rate and direction of flow of oil into the low-pressure, wide-area side of the ram.

The high-pressure hydraulic loop uses water as the working fluid and consists of the small-area ram (ratio of areas is 5.6:1), pressure manifold, dead weight gauge tester, the pressure monitoring transducers mentioned previously, specimen holder, and a third independent transducer whose feedback signals are detected by the Servac and translated into a corrective signal to the servo valve. It is thus possible to maintain a programmed mode of pressurization regardless of the elastic modulus and size of the test specimen, compressibility of the working fluid, and friction caused by the packing glands of the ram. When the difference between the control and feedback signals exceeds a preset value, e.g., at the moment of fracture of the specimen, the hydraulic pressure supply is automatically turned off. A photograph of the testing facility is shown in Fig. 3.

C. Procedure

1. Experimental

Regardless of the type of test, the dead weight gauge tester is used to calibrate the entire system. For measurements of the effect of constant static stress, the valve to the specimen holder is closed and the dead weight tester is loaded with weights corresponding to the desired pressure level. With the function generator in the "ramp" position, the pressure generated by the ram is gradually raised until the weights are lifted and float at a steady level. The pressure control dial of the Servac is locked in this position. Then the ram is commanded to its starting position (zero pressure), the frequency selector is set so as to produce the desired loading rate, and the valve to the dead weight tester is closed and the valve to the specimen holder is opened. Finally, the test cycle is initiated by pressing the appropriate switch. Pressure

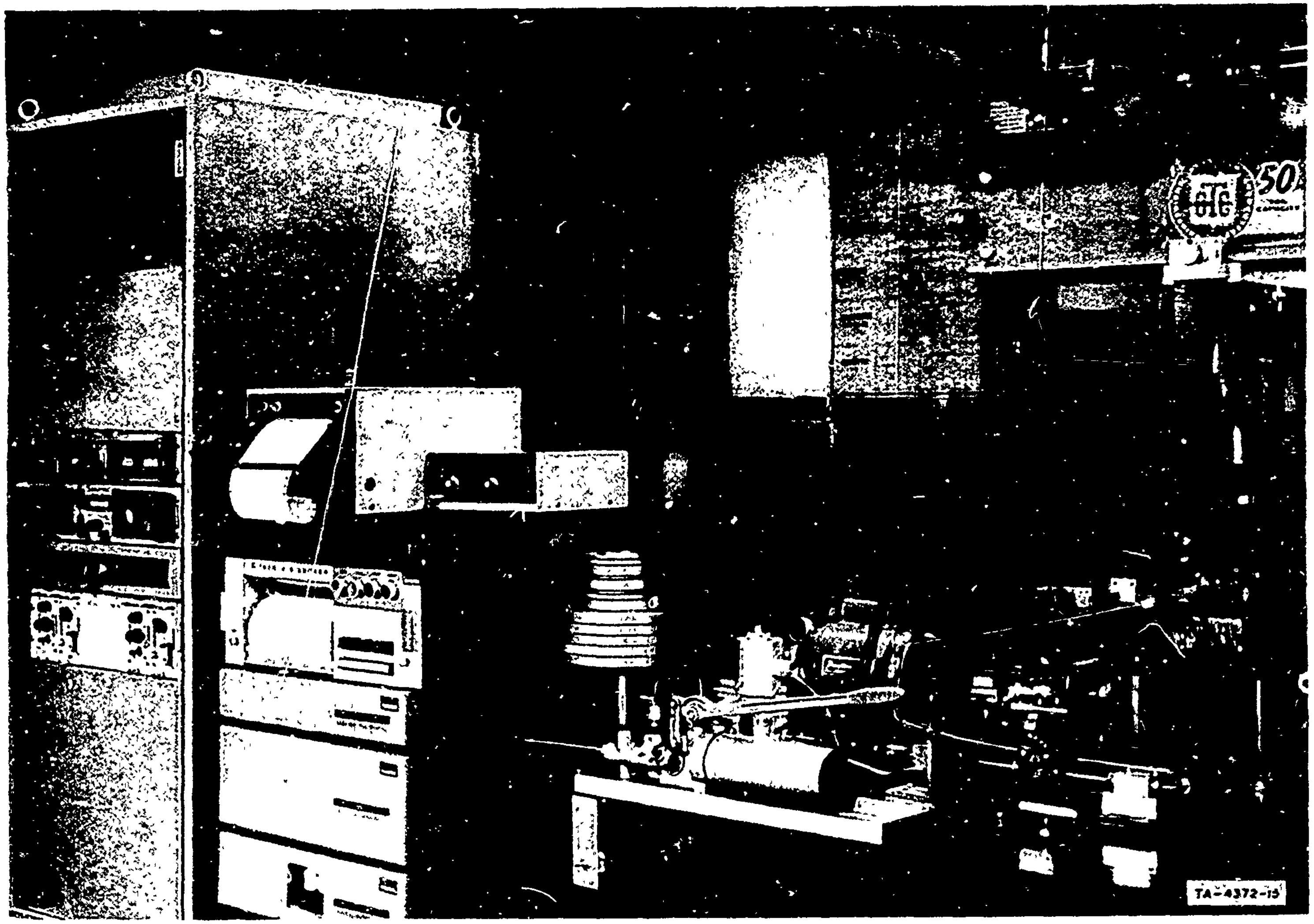


Figure 3. Mechanical Properties Testing Facility.

rises at the predetermined rate, reaches and maintains the desired level, and the time to failure is measured by the length of the straight line on the recorder chart.

The calibration procedure for cyclic fatigue experiments is as follows. With the valve to the specimen holder closed, the upper and lower limits of the desired pressure range are generated by the dead weight gauge tester and are displayed on the recording chart. The range control of the recorder is adjusted so that the lines corresponding to the two pressure limits are as far apart as possible. Then the valve to the dead weight gauge tester is closed, the function generator is put in the sine position, and the frequency selector is adjusted to 4 cps. By manipulating the controls of the Servac, the ram is put into reciprocating motion so that it generates (in the high-pressure loop) a sinusoidal pressure wave whose peaks coincide exactly with the pressure limits established previously by the dead weight gauge tester. To run an experiment, the valve to the specimen holder is opened, the cycle counter is set to zero, and the ram is put into motion. When the specimen breaks, the hydraulic pressure immediately drops to zero, which causes the command signal to turn off the hydraulic system and the counter which indicates the number of cycles to failure. Similarly, when the experiment goes to completion, i.e., when a preselected number of cycles is reached, the hydraulic pressure supply stops automatically.

2. Calculations

Values of maximum tensile stress on the inside walls of test specimens were calculated by using the formula

$$\sigma_{t\ max} = \frac{Pr_i^2}{r_o^2 - r_i^2}\left(1 + \frac{r_o^2}{r_i^2}\right)$$

where P = hydrostatic pressure at fracture (psi)

r_i = internal radius (inches)

r_o = external radius (inches)

Standard deviations (s.d.) were calculated from the formula

$$\text{s.d.} = \sqrt{\frac{\Sigma\, d^2}{n}}$$

where d = deviation from average value of tensile strength

n = number of deviations

Coefficients of variation are given by the formula

$$\frac{\text{standard deviation}}{\text{average utlimate strength}} \times 100$$

IV RESULTS AND DISCUSSION

In order to compare the responses of a given material to various modes of loading, it would be desirable to carry out the entire study on a single lot of test specimens originating in the same batch of raw material and fired at the same time. Unfortunately, the total number of specimens used throughout this entire three-year program was so large that it was not possible to process all of them simultaneously. For this reason, test materials were purchased separately for each phase of the program. The manufacturer's cooperation was obtained and utmost care was exercised to duplicate all processing steps as closely as existing means of control permit. The only known difference between specimens used in the second and third phases of this program is a slight variation in chemical composition which may not be real and may reflect the accuracy of the analytical method used. The microstructure and density--factors known to affect the strength of ceramics--are the same for both lots. It is therefore believed that the lot of specimens evaluated under conditions of cyclic fatigue is essentially the same as the lot used in the study of static fatigue.

For this reason we make the assumption that any similarity or dissimilarity of the effects of cyclic and static fatigue could be attributed to stress conditions only and should not be interpreted as some inherent characteristic of two different ceramic bodies. However, since it was not feasible to make a thorough comparison of all pertinent mechanical properties of the two lots of specimens, the possibility of some subtle property variation cannot be totally discounted.

A. Behavior of Alumina under Constant Static Stress

In the second phase of work, specimens were stressed to various levels representing percentages of "reference-stress," i.e., the average value of ultimate tensile strength (29,600 psi) displayed by the material under conditions of linearly rising stress at the rate of 4,000 psi/sec.

In this year's work the 90% and 80% stress levels were used under identical experimental conditions, without establishing whether the "reference-stress" was the same. It was also planned to use the 70% level after completion of the cyclic study. At that time, however, all specimens had been used up. The average survival time at the 90% level was 7.7 seconds compared to 3.4 seconds a year ago. At the 80% level the average time to failure was 133 seconds compared to 176 seconds last year at the 75% level. The large data scatter and the limited number of samples make it difficult to assess the extent of difference in strength between the two lots of specimens. All individual data are listed in Table A1 of Appendix A. For comparison, static fatigue data obtained in the second phase of work are shown in Appendix B. Also included is the graph of stress rate versus strength on which the choice of the "reference-stress" is based.

B. Behavior of Alumina under Cyclic Tensile Stress

In this study, test specimens were subjected to sinusoidal tensile stresses of various ranges at a frequency of 4 cps and the number of cycles to failure was counted, or, in the case of lower stress levels, the specimens were cycled for a predetermined number of cycles (345,000) taken as an arbitrary fatigue life.

In obtaining the cyclic fatigue data summarized in Table 1, the experimental conditions were arranged so as to permit the generation of three separate S-N curves, varying in the relationship of minimum and maximum stresses employed. This relationship is represented by the stress ratio R, which is the ratio of minimum to maximum stress, i.e., $R = \sigma_{min}/\sigma_{max}$. Three values of R were used--0.14 for Group A, 0.33 for Group B, and 0.78 for Group C. In addition, stress ranges were chosen so that most of the maximum stress values were the same for all three stress ratios.

Individual fatigue data (compiled in Table A2 of Appendix A) best fit a log-normal distribution. For this reason, geometric means rather than arithmetic averages are used in constructing the S-N curves of Figs. 4, 5, and 6. In these graphs maximum stresses are plotted versus

Table 1

BEHAVIOR OF A1-995 ALUMINA UNDER CYCLIC TENSILE STRESS

	σ_{max}	σ_{min}	Mean: Geom. ($\bar{N}$) (cycles)	Mean: Arithm. (cycles)	Mean: $Log_e(\bar{N})$ ($\bar{X}$)	Median (cycles)	Standard Deviation: Arithm. ($\pm$ cycles)	Standard Deviation: Logarithmic (S)	$Log^{-1}(\bar{X} + S)$ (cycles)	$Log^{-1}(\bar{X} - S)$ (cycles)
Group A	27.9	4.0	1.54×10^1	1.87×10^1	1.190	1.70×10^1	1.17×10^1	0.292	3.04×10^1	7.93×10^0
Stress Ratio	23.9	3.4	1.72×10^3	6.55×10^3	3.236	2.93×10^3	1.26×10^4	0.813	1.08×10^4	2.73×10^2
R = 0.14	20.0	2.8	1.16×10^4	5.42×10^4	4.065	8.54×10^4	6.71×10^4	0.980	1.11×10^5	1.21×10^3
	16.3	2.3	*							
	15.8	2.25	**							
Group B	27.9	9.3	1.97×10^1	2.97×10^1	1.294	2.20×10^1	2.45×10^1	0.453	5.58×10^1	6.90×10^0
Stress Ratio	23.9	7.9	3.88×10^3	1.13×10^4	3.589	3.70×10^3	1.58×10^3	0.731	3.03×10^4	4.97×10^2
R = 0.33	20.0	6.6	6.49×10^4	7.70×10^4	4.812	7.78×10^4	4.17×10^4	0.292	1.27×10^5	3.30×10^4
	17.9	6.1	*							
	17.4	5.8	*							
	16.8	5.5	*							
	16.3	5.3	**							
Group C	27.9	21.7	1.80×10^1	2.28×10^1	1.256	1.95×10^1	1.59×10^1	0.321	3.61×10^1	8.61×10^0
Stress Ratio	23.9	18.6	9.88×10^3	3.67×10^4	3.995	2.71×10^4	3.72×10^4	1.078	1.18×10^5	8.25×10^2
R = 0.78	20.0	15.6	*							
	16.8	13.1	**							

* Some experiments were terminated without specimen failure after 345,000 cycles.

** All specimens survived 345,000 cycles.

the number of cycles to failure. The central values are geometric means. They are bracketed on the left by the antilogarithm of the difference of the mean of the logarithms of cycles to failure and the standard logarithmic deviation ($\bar{X}$ - S), and on the right by the antilogarithm of the sum of the same quantities ($\bar{X}$ + S). The S-N curves are drawn to fit the values of geometric means and are extrapolated to the static strength value at 0.25 cycle life (single-stroke tensile strength).

The modified Goodman diagram shown in Fig. 7 is a plot of alternating stress versus mean stress, based on the S-N curves. A family of curves is shown, representing the most probable combinations of stresses for various fatigue lives N. Any value of N can be chosen (within the range of fatigue lives covered by the S-N curves) and the corresponding alternating and mean stresses are derived from the maximum stresses taken from the S-N curves by the relationships $\sigma_a = \sigma_{max}(1 - R)/2$ and $\sigma_m = \sigma_{max}(1 + R)/2$. The line joining the stress combinations at the arbitrary fatigue life of 345,000 cycles intersects the abscissa (where $\sigma_a = 0$) near the 20,000 psi mark. This is somewhat higher than the 24 hours static strength (17,900 psi) determined in the previous study. It may also be noted that the lines representing individual fatigue lives have different slopes and that they are actually not straight lines but tend to be somewhat convex upward. It cannot now be determined whether these phenomena reflect some real property of the test material or are the result of the plotting technique used.

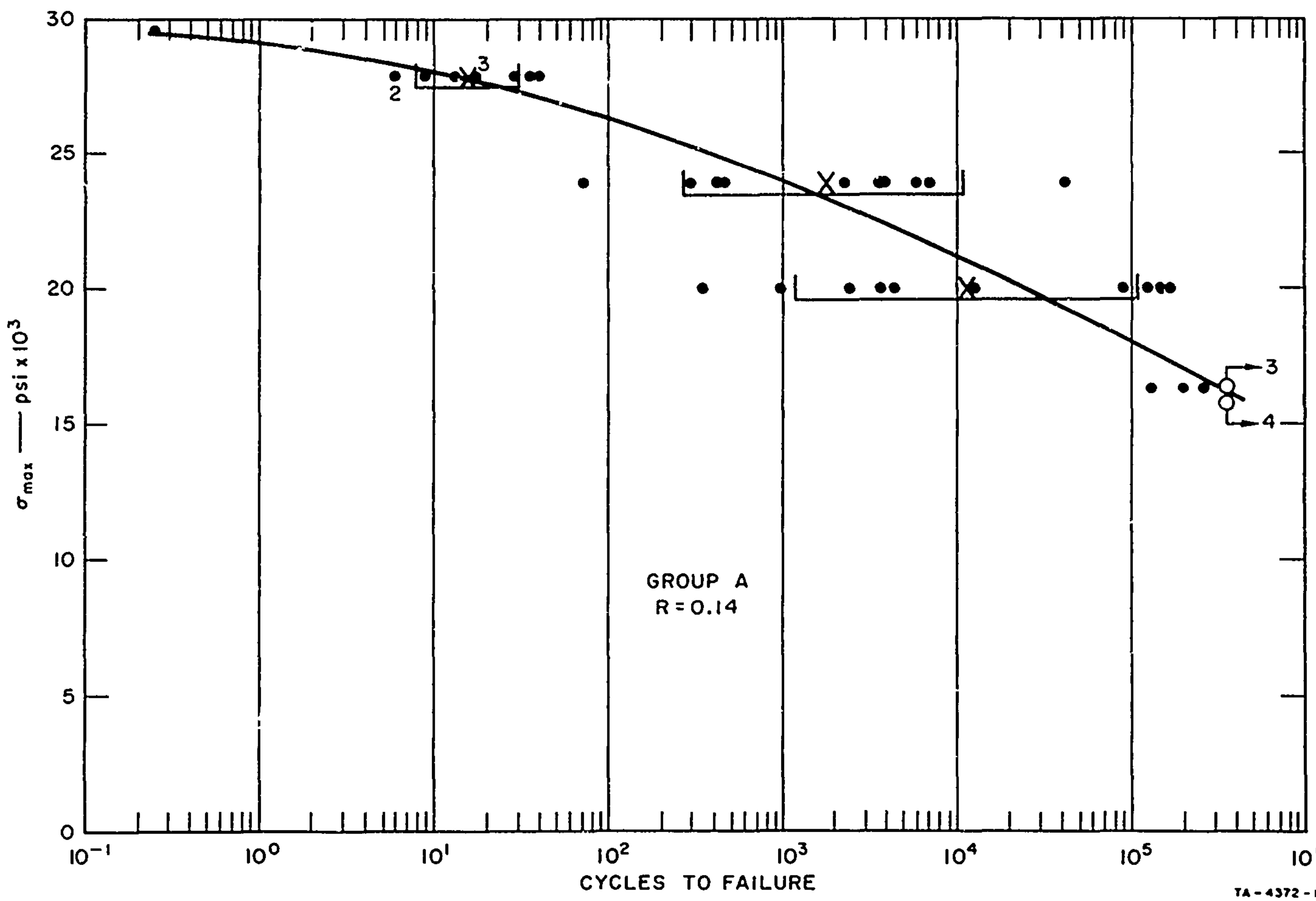


FIG. 4 S-N CURVE FOR AL-995 ALUMINA

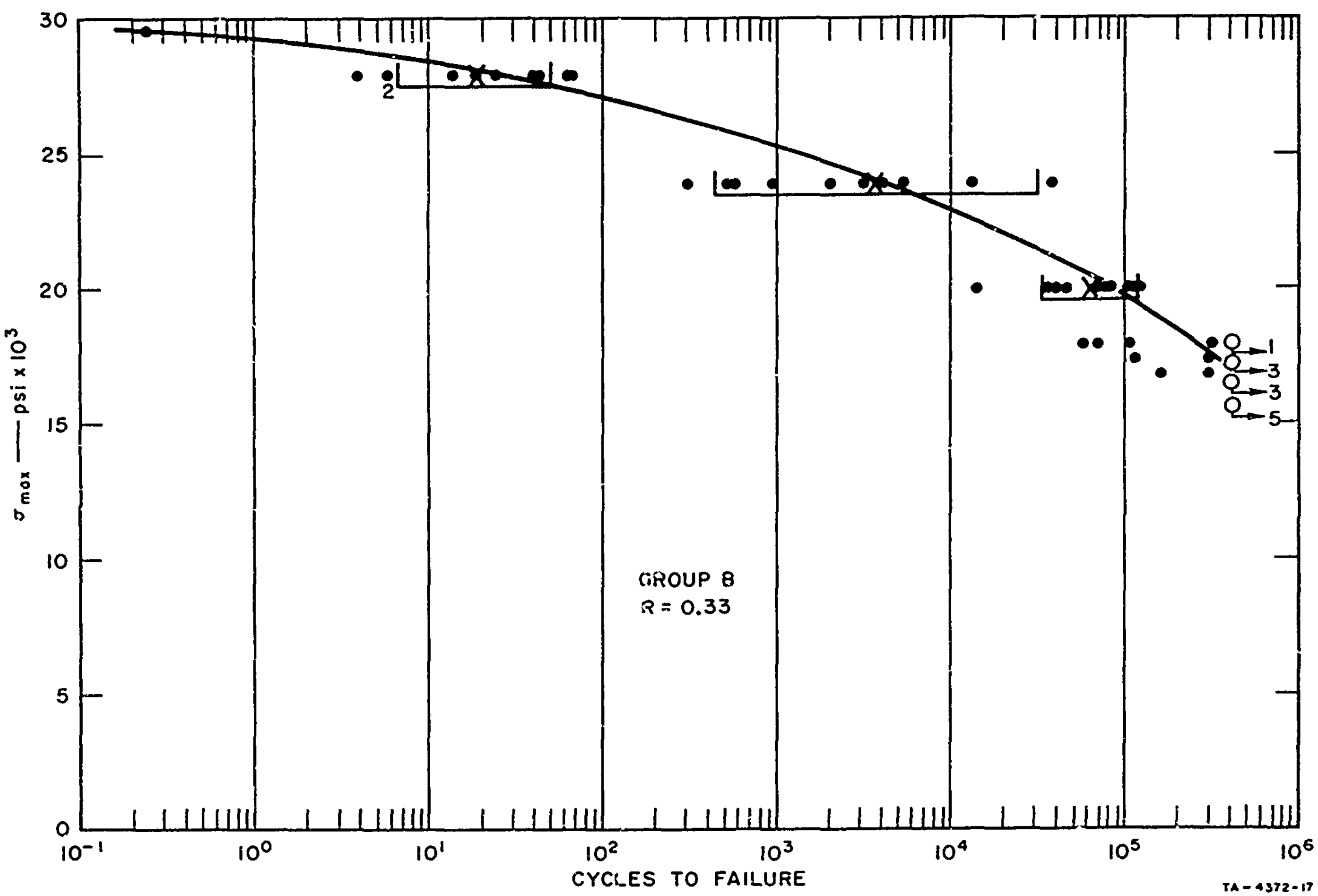


FIG. 5 S-N CURVE FOR AL-995 ALUMINA

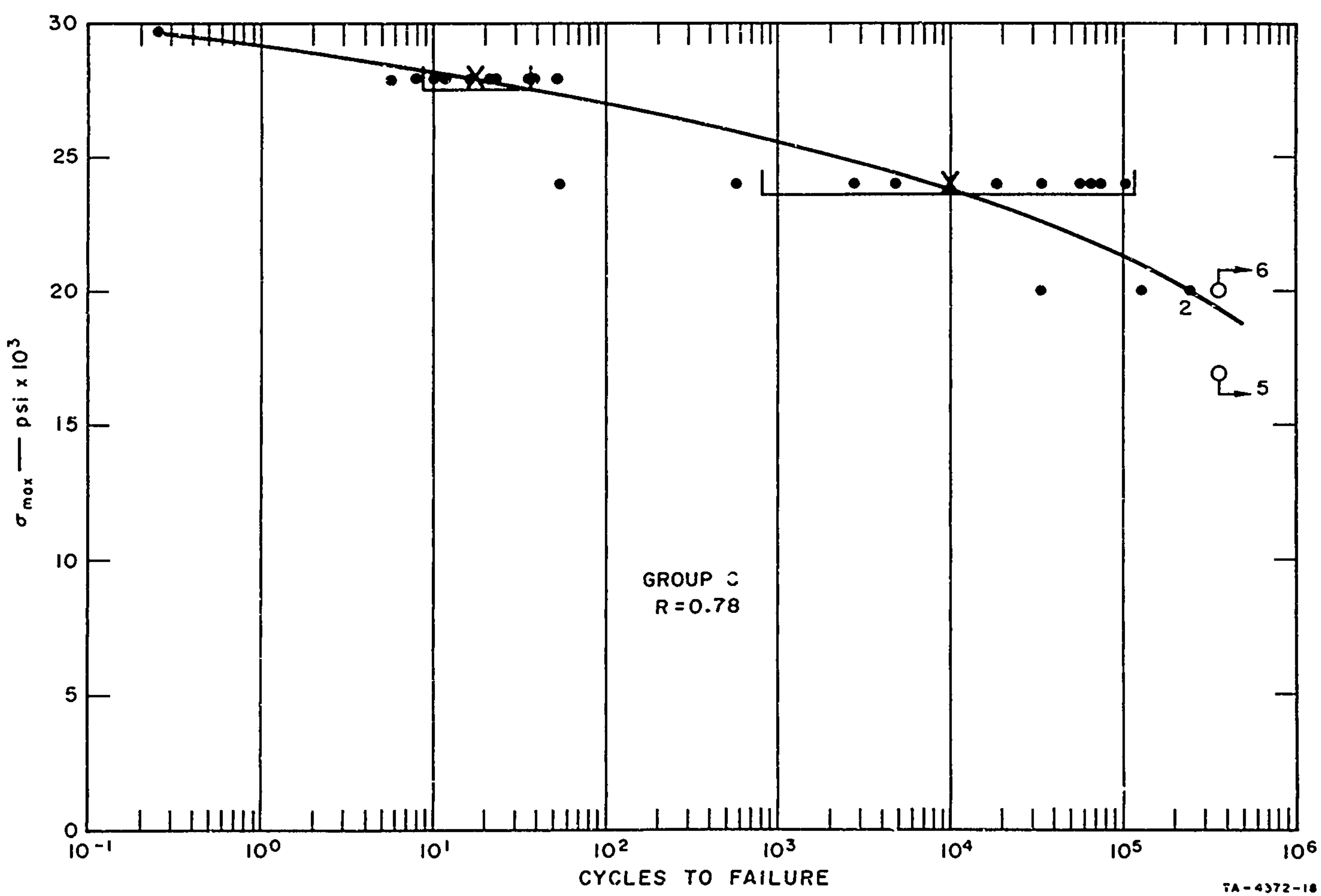


FIG. 6 S-N CURVE FOR AL-995 ALUMINA

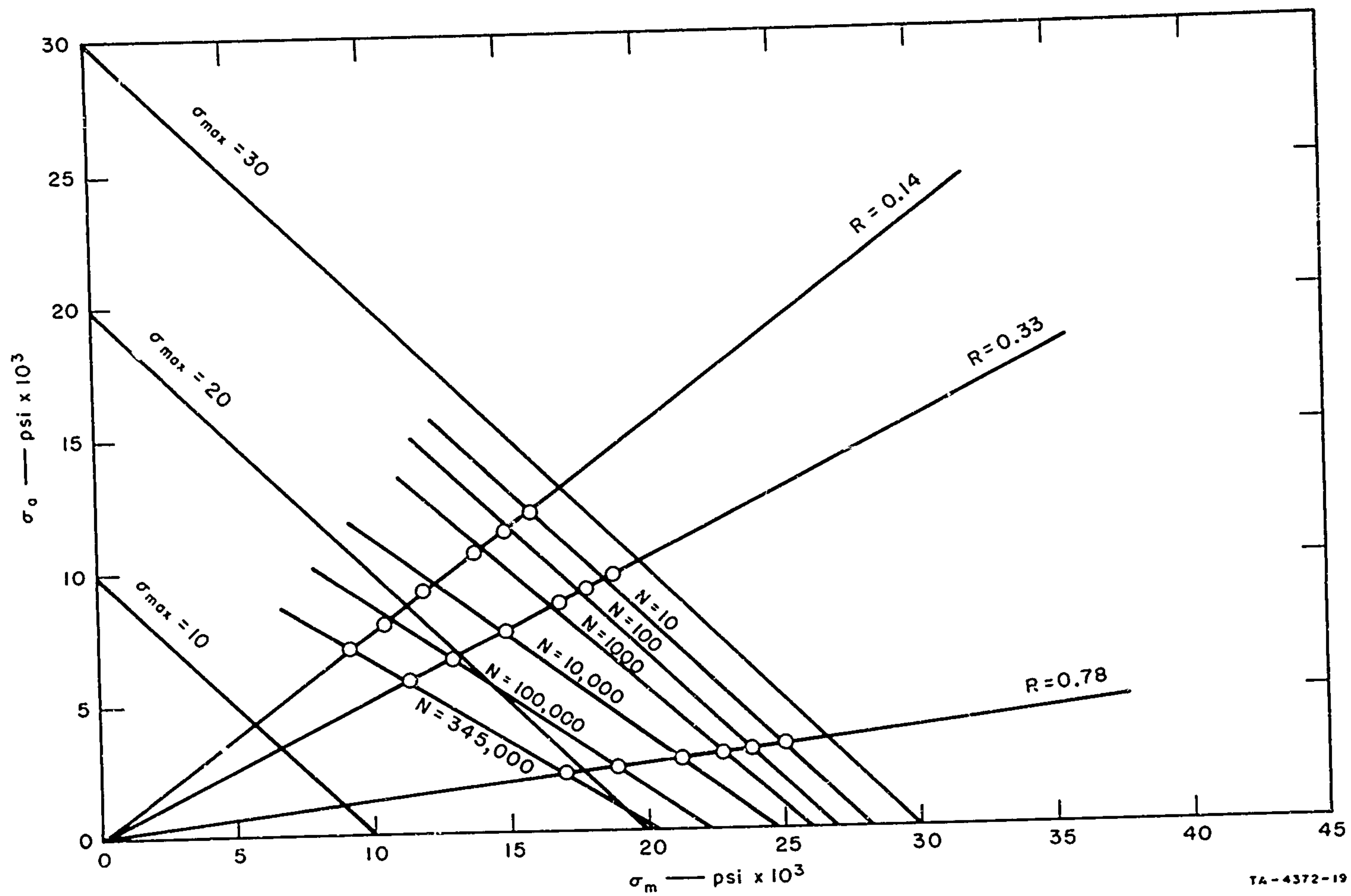


FIG. 7 CONSTANT LIFE FATIGUE DIAGRAM FOR AL-995 ALUMINA

C. Effect of Cyclic Tensile Stress on Ultimate Tensile Strength of Alumina

Specimens which reached the arbitrary limit of 345,000 cycles were subsequently loaded to failure at a stress rate of 10,000 psi/sec. The average ultimate tensile strength of these specimens was 32,900 ± 2,400 psi (7.3%) which is higher by 3,000 psi than the average strength of specimens tested in last year's study of static fatigue. Individual data are compiled in Table A3 and for comparison, last years' data are shown in Table B2.

The reasons for the apparent increase in strength are not clear. The most plausible explanation is that the entire lot of specimens used in the study of cyclic fatigue was somewhat stronger than the specimens used in previous studies. Other indications pointing in the same direction are the slightly longer life under static loads and the fact that the mean stress value at zero alternating stress in Fig. 7 is more than 2,000 psi higher than the experimentally determined maximum static stress at the arbitrary 24 hours limit. Another possibility is that the test material used in the study of cyclic fatigue had a slightly different relationship between strength and stress rate than the material used in the static fatigue study. Ultimate strength data after 24-hour exposure to static stress were obtained at a stress rate of 4,000 psi/sec whereas strength data after cyclic loading were generated at a stress rate of 10,000 psi/sec. It can be seen from Fig. B1 that this difference in loading rate alone does not account for the considerable increase in strength; therefore, the possibility exists that the two lots of test materials exhibited different responses to stress rate.

The strength data in Table A3 show a slightly higher degree of scatter than is normally observed in single-stroke strength measurements on Al-995 alumina. A similar data scatter was also observed during last year's work on the effect of static stresses on strength. The reason for this variability is not known, but it seems to be completely random, and no relationship appears to exist between the ultimate tensile strength and the stress history of individual specimens. The increase of data scatter by prestressing also indicates that proof-testing theories may not be applicable to ceramics.

V CONCLUSIONS

As one would expect, the resistance of alumina to cyclic tensile stresses depends strongly on the applied stress level. A statistical analysis whose results are compiled in Tables C1, C2, C3, and C4 of Appendix C, confirmed that there is a statistically significant correlation between stress level and fatigue life at the 95% confidence level for all test parameters. Although no statistically significant correlation between fatigue life and stress ratio at constant maximum stress can be found, due to small differences in N and to an insufficient number of specimens, the dependence of fatigue life upon stress ratio is demonstrated qualitatively at the lowest level of σ_{max}.

If it is assumed that the rate of crack propagation is solely dependent upon the applied stress level, one would expect the effect of a constant static stress to be more severe than the effect of cyclic stress over the same length of time when σ_{max} of the cyclic stress is the same as the static stress. This is apparently not the case, since in cyclic tension specimens failed at maximum stresses as low as 16,300 psi while no specimens failed at static stresses of 17,900 psi. It is therefore conceivable that in cyclic loading an additional mechanism of crack propagation is active.

The following interpretation of the fracture mechanism is only speculative, and we were unable to find supporting references in literature. It can be visualized that as the crack widens during the first quarter of the cycle, submicroscopic particles become detached from the crack's walls and lodge at other points in the crack. As stress is released and the body contracts, this detritus acts as a fulcrum to keep the crack from closing, thereby generating additional stresses at the tip of the crack. This would explain the effect of stress range on the fatigue life of alumina--i.e., the higher the range, the more damage is

incurred by the body. The postulated mechanism is undoubtedly a time-dependent phenomenon, and as such can be observed only at low stress levels. At higher levels, the principal stress alone is high enough to propagate existing cracks.

For several reasons it is difficult to make a thorough and unambiguous comparison of the various aspects of the mechanical behavior of alumina which were studied under this contract. First, it would have been most desirable to purchase all test specimens at the same time so as to avoid any possibility of slight batch-to-batch variations. Second, the study of cyclic fatigue of alumina is somewhat incomplete because the effect of frequency has not been investigated. It is possible that, in cyclic testing, frequency may play a part similar to that of stress rate in evaluation of ultimate tensile strength. Finally, the time dependence of the strength of ceramics has not been fully appreciated. Stress corrosion, which is known to have a great influence on the strength of glasses, undoubtedly affects ceramics also, although probably to a lesser degree. In this respect it would have been advantageous to carry out all 24-hour experiments under controlled humidity conditions.

VI ACKNOWLEDGMENTS

The writer wishes to express his appreciation to the personnel of the Strength and Dynamics Branch of the Air Force Materials Laboratory for help in the statistical and graphic interpretation of test data. The assistance of Drs. E. G. Chilton, A. V. Fend, and M. J. Pascual of Stanford Research Institute is also acknowledged.

Appendix A

DATA GENERATED IN CURRENT REPORT PERIOD

(February 1965 - February 1966)

Table A1

LOAD-BEARING PERFORMANCE OF ALUMINA
UNDER CONDITIONS OF CONSTANT STATIC TENSILE STRESS

Specimen Number	Loading Rate (psi/sec)	Static Stress (psi)	Percentage of Reference Stress (29,600 psi)*	Time to Failure	
				Minutes	Seconds
1	4×10^3	26,800	90	0	11.2
2	4×10^3	26,800	90	0	8.0
3	4×10^3	26,800	90	0	16.1
4	4×10^3	26,800	90	0	0.3
5	4×10^3	26,800	90	0	2.7
6	4×10^3	26,800	90	0	5.2
7	4×10^3	26,800	90	0	12.0
8	4×10^3	26,800	90	0	1.6
9	4×10^3	26,800	90	0	13.6
10	4×10^3	26,800	90	0	6.7
Average	4×10^3	26,800	90	0	7.7
1	4×10^3	23,900	80	2	52
2	4×10^3	23,900	80	0	34
3	4×10^3	23,900	80	0	36
4	4×10^3	23,900	80	0	30
5	4×10^3	23,900	80	13	11
6	4×10^3	23,900	80	0	13
7	4×10^3	23,900	80	1	37
8	4×10^3	23,900	80	1	24
9	4×10^3	23,900	80	0	39
10	4×10^3	23,900	80	0	36
Average	4×10^3	23,900	80	2	13

* Taken from previous stress rate study.[3]

Table A2

ENDURANCE OF ALUMINA TO CYCLIC TENSILE STRESSES AT 4 cps

GROUP A				
Specimen Number	σ_{max} (psi)	σ_{min} (psi)	R $(\sigma_{min}/\sigma_{max})$	Number of Cycles to Failure
1	27,900	4,000	0.14	9
2	27,900	4,000	0.14	17
3	27,900	4,000	0.14	28
4	27,900	4,000	0.14	34
5	27,900	4,000	0.14	17
6	27,900	4,000	0.14	40
7	27,900	4,000	0.14	13
8	27,900	4,000	0.14	17
9	27,900	4,000	0.14	6
10	27,900	4,000	0.14	6
Average				19
11	23,900	3,400	0.14	3,626
12	23,900	3,400	0.14	41,810
13	23,900	3,400	0.14	304
14	23,900	3,400	0.14	434
15	23,900	3,400	0.14	5,821
16	23,900	3,400	0.14	72
17	23,900	3,400	0.14	6,936
18	23,900	3,400	0.14	468
19	23,900	3,400	0.14	2,225
20	23,900	3,400	0.14	3,776
Average				6,547
21	20,000	2,900	0.14	90,154
22	20,000	2,900	0.14	12,714
23	20,000	2,900	0.14	162,770
24	20,000	2,900	0.14	139,823
25	20,000	2,900	0.14	2,400
26	20,000	2,900	0.14	365
27	20,000	2,900	0.14	992
28	20,000	2,900	0.14	124,334
29	20,000	2,900	0.14	3,626
30	20,000	2,900	0.14	4,357
Average				54,154
31	16,300	2,300	0.14	*
32	16,300	2,300	0.14	128,750
33	16,300	2,300	0.14	*
34	16,300	2,300	0.14	199,870
35	16,300	2,300	0.14	*
36	15,800	2,300	0.14	*
37	15,800	2,300	0.14	*
38	15,800	2,300	0.14	*
39	15,800	2,300	0.14	*

* Specimen reached arbitrary fatigue life of 345,000 cycles without failure.

Table A2 (Continued)

ENDURANCE OF ALUMINA TO CYCLIC TENSILE STRESSES AT 4 cps

GROUP B				
Specimen Number	σ_{max} (psi)	σ_{min} (psi)	R $\left(\sigma_{min}/\sigma_{max}\right)$	Number of Cycles to Failure
1	27,900	9,300	0.33	15
2	27,900	9,300	0.33	6
3	27,900	9,300	0.33	45
4	27,900	9,300	0.33	25
5	27,900	9,300	0.33	69
6	27,900	9,300	0.33	6
7	27,900	9,300	0.33	4
8	27,900	9,300	0.33	19
9	27,900	9,300	0.33	67
10	27,900	9,300	0.33	41
Average				30
11	23,900	7,900	0.33	6,624
12	23,900	7,900	0.33	39,015
13	23,900	7,900	0.33	3,285
14	23,900	7,900	0.33	321
15	23,900	7,900	0.33	41,109
16	23,900	7,900	0.33	532
17	23,900	7,900	0.33	2,133
18	23,900	7,900	0.33	978
19	23,900	7,900	0.33	4,120
20	23,900	7,900	0.33	15,220
Average				11,138
21	20,000	6,600	0.33	37,490
22	20,000	6,600	0.33	111,120
23	20,000	6,600	0.33	48,086
24	20,000	6,600	0.33	83,652
25	20,000	6,600	0.33	129,352
26	20,000	6,600	0.33	87,853
27	20,000	6,600	0.33	16,057
28	20,000	6,600	0.33	41,992
29	20,000	6,600	0.33	71,954
30	20,000	6,600	0.33	142,093
Average				76,965

Table A2 (Continued)

ENDURANCE OF ALUMINA TO CYCLIC TENSILE STRESSES AT 4 cps

GROUP B (Concluded)				
Specimen Number	σ_{max} (psi)	σ_{min} (psi)	R $(\sigma_{min}/\sigma_{max})$	Number of Cycles to Failure
31	17,900	6,100	0.33	*
32	17,900	6,100	0.33	107,374
33	17,900	6,100	0.33	58,200
34	17,900	6,100	0.33	70,992
35	17,900	6,100	0.33	333,480
36	17,400	5,800	0.33	306,876
37	17,400	5,800	0.33	*
38	17,400	5,800	0.33	*
39	17,400	5,800	0.33	126,540
40	17,400	5,800	0.33	*
41	16,800	5,500	0.33	304,950
42	16,800	5,500	0.33	183,600
43	16,800	5,500	0.33	*
44	16,800	5,500	0.33	*
45	16,800	5,500	0.33	*
46	16,300	5,300	0.33	*
47	16,300	5,300	0.33	*
48	16,300	5,300	0.33	*
49	16,300	5,300	0.33	*
50	16,300	5,300	0.33	*

* Specimen reached arbitrary fatigue life of 345,000 cycles without failure.

Table A2 (Concluded)

ENDURANCE OF ALUMINA TO CYCLIC TENSILE STRESSES AT 4 cps

GROUP C				
Specimen Number	σ_{max} (psi)	σ_{min} (psi)	R $(\sigma_{min}/\sigma_{max})$	Number of Cycles to Failure
1	27,900	21,700	0.78	8
2	27,900	21,700	0.78	53
3	27,900	21,700	0.78	11
4	27,900	21,700	0.78	38
5	27,900	21,700	0.78	23
6	27,900	21,700	0.78	17
7	27,900	21,700	0.78	10
8	27,900	21,700	0.78	6
9	27,900	21,700	0.78	22
10	27,900	21,700	0.78	40
Average				23
11	23,900	18,600	0.78	60,049
12	23,900	18,600	0.78	4,680
13	23,900	18,600	0.78	74,112
14	23,900	18,600	0.78	585
15	23,900	18,600	0.78	56
16	23,900	18,600	0.78	2,778
17	23,900	18,600	0.78	102,924
18	23,900	18,600	0.78	18,907
19	23,900	18,600	0.78	35,383
20	23,900	18,600	0.78	67,971
Average				36,745
21	20,000	15,600	0.78	33,350
22	20,000	15,600	0.78	254,895
23	20,000	15,600	0.78	*
24	20,000	15,600	0.78	*
25	20,000	15,600	0.78	*
26	20,000	15,600	0.78	*
27	20,000	15,600	0.78	256,070
28	20,000	15,600	0.78	*
29	20,000	15,600	0.78	127,910
30	20,000	15,600	0.78	*
31	16,800	13,100	0.78	*
32	16,800	13,100	0.78	*
33	16,800	13,100	0.78	*
34	16,800	13,100	0.78	*
35	16,800	13,100	0.78	*

* Specimen reached arbitrary fatigue life of 345,000 cycles without failure.

Table A3

EFFECT OF CYCLIC TENSILE STRESS ON THE ULTIMATE TENSILE STRENGTH OF ALUMINA

Specimen Group and Number	Loading Rate (psi/sec)	Ultimate Tensile Strength (psi)	Deviation (psi)
A 31	10^4	31,300	- 1600
A 33	10^4	32,600	- 300
A 35	10^4	35,900	+ 3000
A 36	10^4	30,600	- 2300
A 37	10^4	28,500	- 4400
A 38	10^4	33,800	+ 900
A 39	10^4	36,200	+ 3300
A 40	10^4	*	
B 31	10^4	29,500	- 3400
B 37	10^4	32,300	- 600
B 38	10^4	33,800	+ 900
B 40	10^4	31,600	- 1300
B 43	10^4	33,500	+ 600
B 44	10^4	37,000	+ 4100
B 45	10^4	32,100	- 800
B 46	10^4	33,500	+ 600
B 47	10^4	33,800	+ 900
B 48	10^4	32,800	- 100
B 49	10^4	30,600	- 2300
B 50	10^4	34,800	+ 1900
C 23	10^4	34,400	+ 1500
C 24	10^4	34,300	+ 1400
C 25	10^4	30,100	- 2800
C 26	10^4	34,300	+ 1400
C 28	10^4	37,100	+ 4200
C 30	10^4	38,200	- 4700
C 31	10^4	33,400	+ 500
C 32	10^4	28,000	- 4900
C 33	10^4	33,900	+ 1000
C 34	10^4	34,100	+ 1200
C 35	10^4	34,300	+ 1400

Note:

Average ultimate tensile strength = 32,900 psi
Standard deviation = $\pm$ 2400 psi
Coefficient of variation = 7.3%

* Specimen broken in handling

Appendix B

DATA GENERATED IN PREVIOUS REPORT PERIOD

(January 1964 - January 1965)

Table B1

LOAD-BEARING PERFORMANCE OF ALUMINA
UNDER CONDITIONS OF CONSTANT STATIC TENSILE STRESS

Specimen Number	Loading Rate (psi/sec)	Static Stress (psi)	Percentage of Reference Stress (29,600 psi)	Time to Failure		
				Hour	Minute	Seconds
1	4×10^3	26,800	90	0	0	4.7
2	4×10^3	26,800	90	0	0	4.8
3	4×10^3	26,800	90	0	0	1.7
4	4×10^3	26,800	90	0	0	0.2
5	4×10^3	26,800	90	0	0	3.3
6	4×10^3	26,800	90	0	0	6.4
7	4×10^3	26,800	90	0	0	0.0
8	4×10^3	26,800	90	0	0	0.5
9	4×10^3	26,800	90	0	0	1.0
10	4×10^3	26,800	90	0	0	21.3
Average	4×10^3	26.800	90			4.4
1	4×10^3	22,400	75	0	10	58
2	4×10^3	22,400	75	0	0	11
3	4×10^3	22,400	75	0	6	11
4	4×10^3	22,400	75	0	3	[illegible]0
5	4×10^3	22,400	75	0	1	20
6	4×10^3	22,400	75	0	0	41
7	4×10^3	22,400	75	0	1	42
8	4×10^3	22,400	75	0	1	58
9	4×10^3	22,400	75	0	1	34
10	4×10^3	22,400	75	0	1	41
Average	4×10^3	22,400	75		2	56
1	4×10^3	20,800	70	0	29	31
2	4×10^3	20,800	70	0	3	8
3	4×10^3	20,800	70	3	15	20
4	4×10^3	20,800	70	6	15	20
5	4×10^3	20,800	70	12	43	45
6	4×10^3	20,800	70	0	1	30
7	4×10^3	20,800	70	0	5	50
8	4×10^3	20,800	70	0	1	0
9	4×10^3	20,800	70	10	27	29
10	4×10^3	20,800	70	0	38	48
Average	4×10^3	20,800	70	3	23	0

Table B2

EFFECT ON ULTIMATE TENSILE STRENGTH OF Al-995 ALUMINA OF 24-HOUR EXPOSURE TO A STATIC TENSILE STRESS OF 17,900 PSI

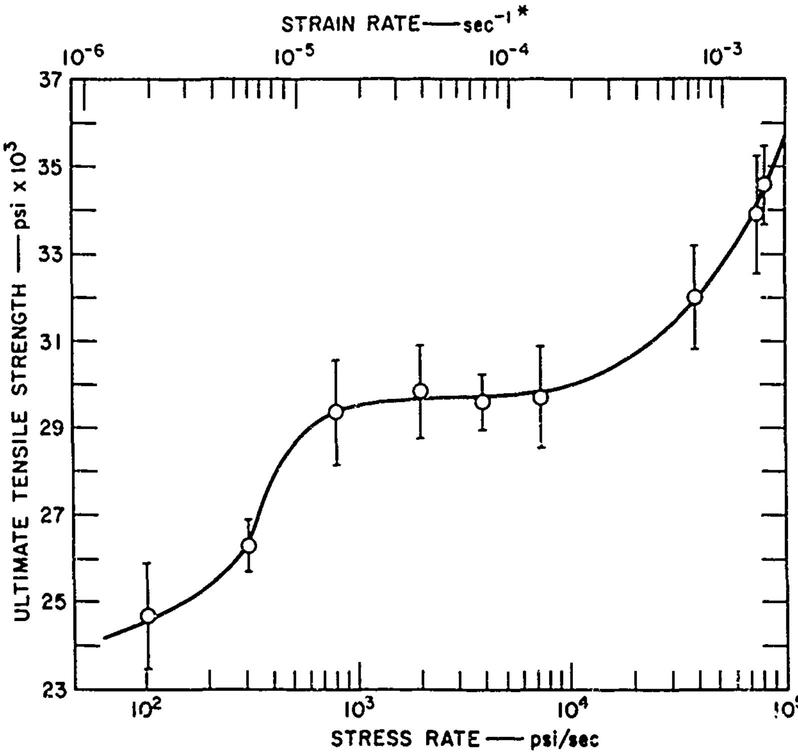
Specimen Number	Loading Rate (psi/sec)	Ultimate Tensile Strength (psi)	Deviation (psi)
1	4×10^3	31,500	+ 1600
2	4×10^3	26,700	- 3200
3	4×10^3	30,600	+ 700
4	4×10^3	29,100	- 800
5	4×10^3	26,700	- 3200
6	4×10^3	28,800	- 1100
7	4×10^3	30,900	+ 1000
8	4×10^3	30,800	+ 900
9	4×10^3	31,100	+ 1200
10	4×10^3	32,600	+ 2700

Note:

Average ultimate tensile strength = 29,900 psi

Standard deviation = $\pm$ 2,000 psi

Coefficient of variation = 6.7%



*CALCULATED FROM STRESS RATE USING A MODULUS OF 53×10^6 psi

TA-4372-2R

FIG. B-1 EFFECT OF STRESS RATE ON ULTIMATE TENSILE STRENGTH OF AL-995 ALUMINA

Appendix C

STATISTICAL ANALYSIS OF DATA

The following analyses were performed to determine how the logarithms of cycles to failure (y) depend upon the variables σ_{max}, σ_{min}, and $R = \sigma_{min}/\sigma_{max}$. Since (y) may have come from a normal distribution, an analysis of variance was used and a statistical test was applied in which the null hypothesis that the samples came from the same normal population was evaluated. Also, a statistical test which did not assume normality or any other type of distribution for the parent population was applied. To get an idea of the relative strength of the dependence of (y) upon the three independent variables σ_{max}, σ_{min}, and R, linear regressions were obtained in which various combinations of variables were included.

Table C1

SETS OF SAMPLES ANALYZED

S_{1A} ≡ Group A specimens	1-10
S_{2A} ≡ Group A specimens	11-20
S_{3A} ≡ Group A specimens	21-30
S_{1B} ≡ Group B specimens	1-10
S_{2B} ≡ Group B specimens	11-20
S_{3B} ≡ Group B specimens	21-30
S_{1C} ≡ Group C specimens	1-10
S_{2C} ≡ Group C specimens	11-20
S_{3C} ≡ Group C specimens	21-30

Table C2

MEANS AND VARIANCES OF LOGARITHMS OF CYCLES TO FAILURE

R	σ_{max} (psi)		
	27,900	23,900	20,000
.14	1.18965 .08423	3.23590 .66047	4.06485 .96015
.33	1.29421 .20517	3.48919 .534[illegible]0	4.81182 .08544
.78	1.25634 .10307	3.99480 1.16298	5.36716 .10646

Table C2 gives the means of the logarithms of cycles to failure (y) and the variances of (y) for the three levels of σ_{max} and R. Looking at the means (upper entries) we notice a definite increase as σ_{max} is decreased. Similarly, but less pronounced, there is an increase in (y) as R is increased. This indicates that the mean logarithm of cycles to failure (y) is strongly dependent upon σ_{max} and only slightly dependent upon R.

Table C3 shows the results of applying the F test to the sets shown in the left column. This test is used to decide whether differences among the means of the sets considered may be attributed to chance or to the fact that they do not come from the same normal population. As can be seen from the table, when σ_{max} changes as in the first three sets, the null hypothesis must be rejected, indicating that (y) is strongly dependent upon σ_{max}. For R varying as in the last three groups but σ_{max} remaining constant, we cannot reject the null hypothesis until we get to the set S_{3A}, S_{3B}, S_{3C} in which the value of σ_{max} is the least--20,000 psi. Notice that as σ_{max} decreases, F gets larger, indicating a stronger dependence upon R of the log of cycles to failure at the lower σ_{max} stress levels.

Table C3

F-TEST*

Sets Being Considered	F (calc.)	F(table) Critical Value 5% level	Decision on Null Hypoth.
S_{1A}, S_{2A}, S_{3A}	38.54	3.35	reject
S_{1B}, S_{2B}, S_{3B}	114.15	3.35	reject
S_{1C}, S_{2C}, S_{3C}	94.65	3.35	reject
S_{1A}, S_{1B}, S_{1C}	0.21	3.35	cannot reject
S_{2A}, S_{2B}, S_{2C}	1.83	3.35	cannot reject
S_{3A}, S_{3B}, S_{3C}	11.12	3.35	reject

* Null hypothesis: the samples are from the same normal population.

Table C4 shows the results of applying the nonparametric Mann-Whitney test to the sets shown in the lefthand column. This test is used to decide whether the rank ordering which occurs when the two sets are pooled into a single set could have occurred by chance alone if each sample was drawn from the same parent population. Normality of the parent population is not assumed in this test as it is in the F test. In the first six cases in which R is fixed but σ_{max} is changing, the null hypothesis of identical parent distributions must be rejected in all cases but the pair S_{2A} and S_{3A}. For the second group in which σ_{max} remains constant but R changes, rejection is warranted only in the cases when S_{3A} versus S_{3C} and S_{3B} versus S_{3C} are compared, i.e., where σ_{max} has its smallest value (20,000 psi). This is again an indication that (y) is more dependent upon R at the lower maximum stress level.

Table C5 shows the results of applying the Mann-Whitney U test to the sets in the lefthand column. Here all of the first 30 specimens of one group are compared with the first 30 specimens of another group. The decision is that the null hypothesis of identical parent populations cannot be rejected in any case at the 5% level and in only one case at the 10% level. When sets which failed at a higher maximum stress level are combined with those which failed at the lower stress level, the dependence of (y) upon R at the lower stress level is hidden and shows only where the largest change in R takes place, between Groups A and C.

Table C4

MANN-WHITNEY TEST*

	Sets Being Compared	U (calc.)	5% Significance Level U(table) Critical Value	5% Significance Level Decision on Null Hypoth.	10% Significance Level U(table) Critical Value	10% Significance Level Decision on Null Hypoth.
← R not changing →	S_{1A} & S_{2A}	0	23	reject	27	reject
	S_{1A} & S_{3A}	0	23	reject	27	reject
	S_{2A} & S_{3A}	27.5	23	cannot reject	27	cannot reject
	S_{1B} & S_{2B}	0	23	reject	27	reject
	S_{1B} & S_{3B}	0	23	reject	27	reject
	S_{2B} & S_{3B}	4	23	reject	27	reject
	S_{1C} & S_{2C}	0	23	reject	27	reject
	S_{1C} & S_{3C}	0	23	reject	27	reject
	S_{2C} & S_{3C}	5	23	reject	27	reject
← σ_{max} not changing →	S_{1A} & S_{1B}	39	23	cannot reject	27	cannot reject
	S_{1A} & S_{1C}	43	23	cannot reject	27	cannot reject
	S_{1B} & S_{1C}	45	23	cannot reject	27	cannot reject
	S_{2A} & S_{2B}	39	23	cannot reject	27	cannot reject
	S_{2A} & S_{2C}	32	23	cannot reject	27	cannot reject
	S_{2B} & S_{2C}	32	23	cannot reject	27	cannot reject
	S_{3A} & S_{3B}	34	23	cannot reject	27	cannot reject
	S_{3A} & S_{3C}	6	23	reject	27	reject
	S_{3B} & S_{3C}	10	23	reject	27	reject

* Null Hypothesis: the samples come from the same parent population.

Table C5

MANN-WHITNEY TEST

Sets Being Compared	Z (Calc.)	Z(table) Critical Value	Decision on Null Hypoth.	Z(table) Critical Value	Decision on Null Hypoth.
A & B	1.02	1.96	cannot reject	1.65	cannot reject
A & C	1.80	1.96	canrot reject	1.65	reject
B & C	1.03	1.96	car¬ot reject	1.65	cannot reject

The following linear regressions were obtained by using all data in which the specimens failed, (y) is in natural logarithms.

First, using all the variables σ_{max}, σ_{min}, and R,

$$(y) = 22.543 - (6.799 \times 10^{-4}\ \sigma_{max}) - (6.848 \times 10^{-4}\ \sigma_{min}) + 18.89\ R \qquad (1)$$

with a value of the square of the correlation coefficient of 0.812. This shows that about 81% of the variance in the data is accounted for by the relationship given in (1).

Next, using σ_{max} only,

$$(y) = 28.358 - 8.876 \times 10^{-4}\ \sigma_{max} \qquad (2)$$

with a value of the square of the correlation coefficient of 0.77. Hence with σ_{max} alone the linear form accounts for about 77% of the variance.

Next, using σ_{max} and σ_{min},

$$(y) = 28.830 - (9.381 \times 10^{-4}\ \sigma_{max}) + (7.843 \times 10^{-5}\ \sigma_{min}) \qquad (3)$$

with a value of 0.787 for the square of the correlation coefficient.

Hence including σ_{min} increases the range of variation accounted for by about 2%. This indicates that the dependence of the log cycles to failure upon σ_{min} at stress levels involved is practically negligible.

Next, using σ_{max} and R,

$$(y) = 28.159 - (9.150 \times 10^{-4} \sigma_{max}) + 2.217 R \quad (4)$$

with a value of 0.792 for the square of the correlation coefficient. Hence including R at these stress levels increases the range of variation accounted for by about 2%, so that the dependence of (y) upon R is also practically negligible.

Since it was suspected that R may be a more significant variable at the lower maximum stress levels, a linear regression equation for (y) was obtained as a linear function of R only at the lower maximum stress level of 20,000 psi. The result is

$$(y) = 9.119 + 4.353 R \quad (5)$$

with a value of 0.408 for the square of the correlation coefficient.

Using all stress levels and R as the only dependent variable gave

$$(y) = 7.617 - 0.272 R \quad (6)$$

with a value of 0.0003 for the square of the correlation coefficient. Again the indication is that (y) is more dependent upon R at the lower maximum stress levels.

The conclusions reached from the analyses can be listed as follows:

1. The distribution of the logarithms of the cycles to failure at the stress levels considered differs significantly for different values of σ_{max}. This is established by both of the statistical tests applied.
2. The distribution of the logarithms of the cycles to failure at the stress levels considered does not differ significantly for different values of σ_{min} or R when σ_{max} is held constant, except at

the lowest value of σ_{max} (20,000 psi). This is established by both of the statistical tests applied.

3. The relative importance of the variables σ_{max}, σ_{min}, and R at the stress levels considered is shown by the regression analysis. By using only σ_{max} as the independent variable and obtaining the least-squares linear relationship between (y) and σ_m, we account for 77% of the variation in (y). By introducing σ_{min} as a second variable we only account for about 2% more of the variation. This gives a measure of how much more important σ_{max} is than either R or σ_{min}, when all stress levels for σ_{max} are considered.

4. The stronger dependence upon R at lower values of σ_{max} was shown by obtaining (y) as a linear function of R only, at the constant maximum stress level of 20,000 psi.

5. It appears that as σ_{max} is lowered there will be a crossover point at which R will become the more significant variable. This maximum stress level could be found only through more experimentation at lower maximum stress levels.

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13. ABSTRACT The cyclic fatigue behavior of a high-alumina body was investigated under conditions of cyclic tensile stress by using hydraulically expanded cylindrical specimens. Experimental results show that the fatigue strength of alumina decreases with increasing maximum stress and is influenced by the value of the stress ratio. Specimens which reached the arbitrary fatigue life of 24 hours (345,000 cycles) were subsequently stressed to failure at a rate of 10^4 psi/sec. Their average ultimate tensile strength was 32,800 psi. indicating that the cyclic stress conditions under which that fatigue life was reached did not weaken the test material and could probably be sustained indefinitely.

S-N curves and a constant life fatigue diagram for the alumina body studied were established.

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